M 2 ti-Party Computation
Part 1

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Plan for the next 3 hours…

- **Part 1: Secure Computation with a Trusted Dealer**
  - Warmup: One-Time Truth Tables
  - Evaluating Circuits with Beaver’s trick
  - MAC-then-Compute for Active Security
- **Part 2: Oblivious Transfer**
  - OT: Definitions and Applications
  - Passive Secure OT Extension
  - OT Protocols from DDH (Naor-Pinkas/PVW)
- **Part 3: Garbled Circuits**
  - GC: Definitions and Applications
  - Garbling gate-by-gate: Basic and optimizations
  - Active security 101: simple-cut-and choose, dual-execution

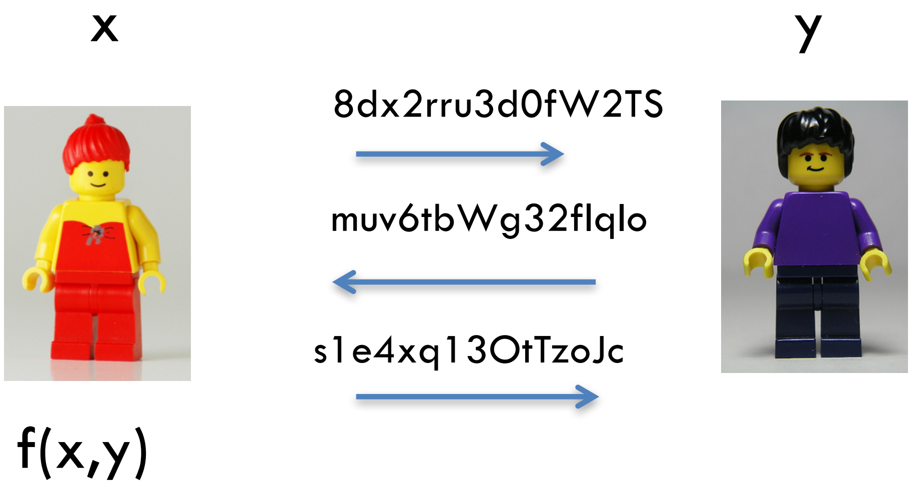
Want more?

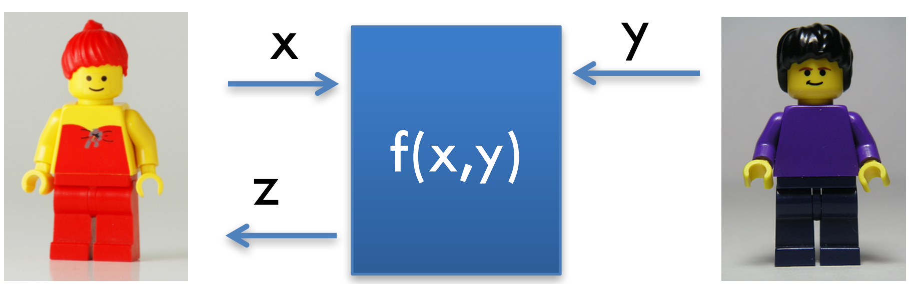
- **Cryptographic Computing – Foundations**
  - http://orlandi.dk/crycom
  - Programming & Theory Exercises
  - Will be happy to answer questions by mail!

…also the reason why I cannot stay here longer ☹

- **These slides** (+ references & pointers)
  - http://orlandi.dk/ecrypt

Secure Computation



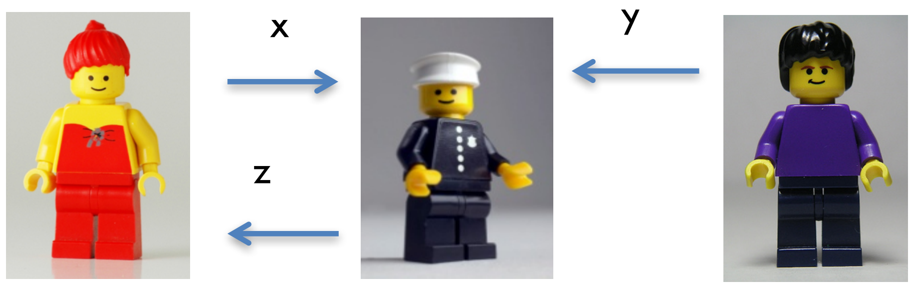


- *Privacy*
- *Correctness*
- *Input independence*
- …

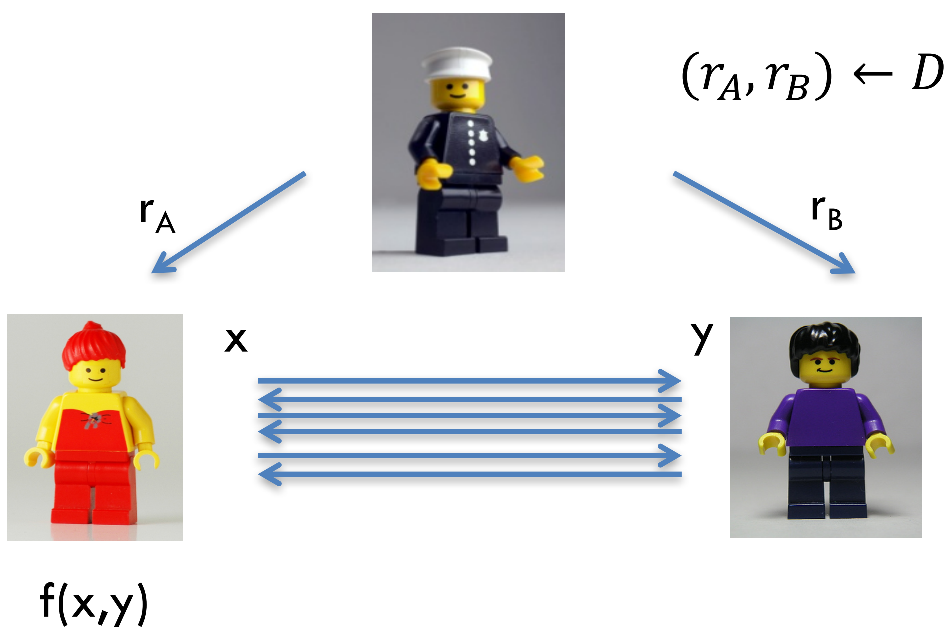
What kind of *Secure* Computation?

- ***Dishonest majority***
  - The adversary can corrupt up to n-1 participants (n=2).
- ***Static Corruptions***
  - The adversary chooses which party is corrupted before the protocol starts.
- ***Passive & Active Corruptions***
  - Adversary follows the protocol vs.
    (aka *semi-honest, honest-but-curious*)
  - Adversary can behave arbitrarily
    (aka *malicious, byzantine*)
- ***No guarantees of fairness or termination***
  - Security with abort

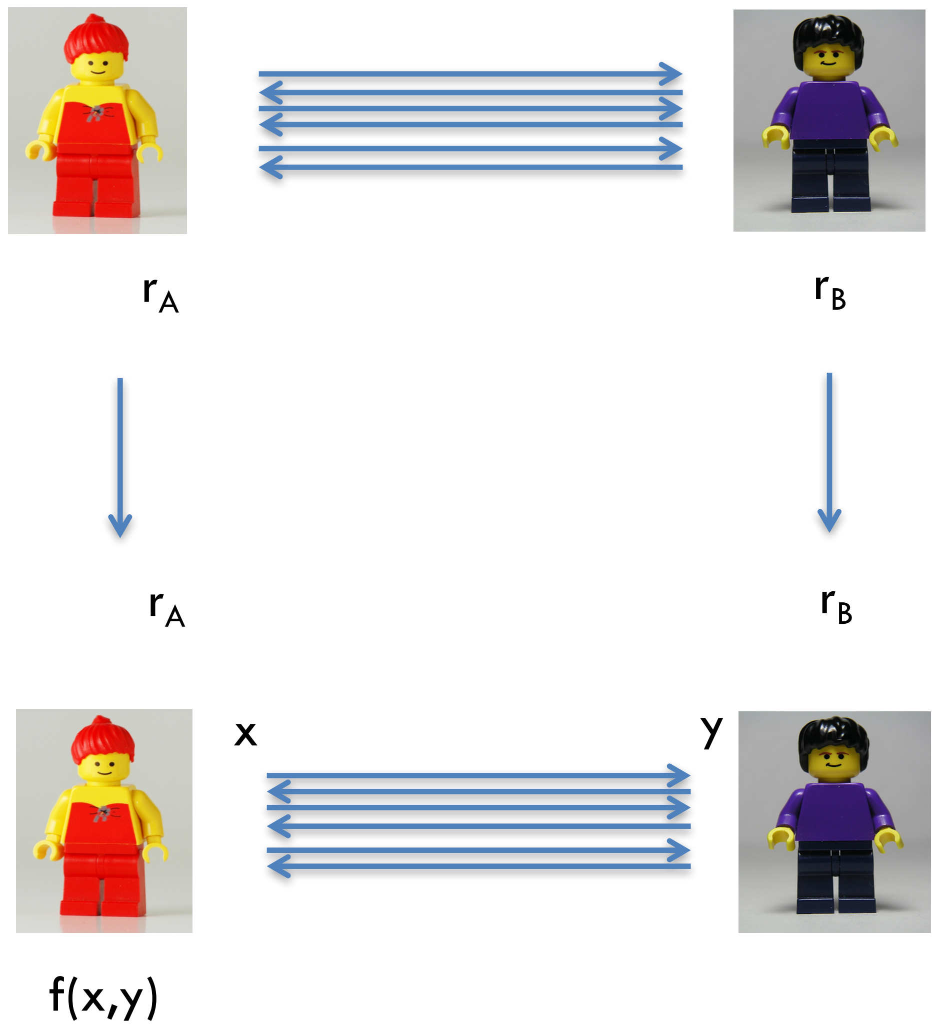
Trusted Party



Trusted Dealer



Preprocessing



- Independent of ***x,y***
- Tipically only depends on ***size of f***
- Uses public key crypto technology ***(slower)***

Online Phase

- Uses only information theoretic tools ***(order of magn. faster)***

Part 1: Secure Computation with a Trusted Dealer

- **Warmup: One-Time Truth Tables**
- Evaluating Circuits with Beaver’s trick
- MAC-then-Compute for Active Security

"The simplest 2PC protocol ever"

$(r_A, r_B) \leftarrow D$

r_A

r_B

x

y

f(x,y)

“The simplest 2PC protocol ever” OTTT (Preprocessing phase)

1) Write the truth table of the function F you want to compute

y

x		0	1	2	3
	0	3	2	2	2
	1	3	0	0	4
	2	1	0	0	4
	3	1	1	4	4

“The simplest 2PC protocol ever” OTTT (Preprocessing phase)

2) Pick random (r, s), rotate rows and columns

s=3

r=1

	0	1	2	3
0	1	4	4	1
1	2	2	2	3
2	0	0	4	3
3	0	0	4	1

“The simplest 2PC protocol ever” OTTT (Preprocessing phase)

3) Secret share the truth table i.e.,

Pick T1 at random, and let

T2 =

1	4	4	1
2	2	2	3
0	0	4	3
0	0	4	1

- T1

“The simples… (…

“Privacy”:
inputs masked w/uniform random values

T1 , r

T2 , s

$u = x + r$ →

← $v = y + s$

← $T2[u,v]$

Correctness:
by construction

output $f(x,y) = T1[u,v] + T2[u,v]$

What about active security?

T1 , r

T2 , s

$u = x + r$ →

$v = y + s + e1$ ←

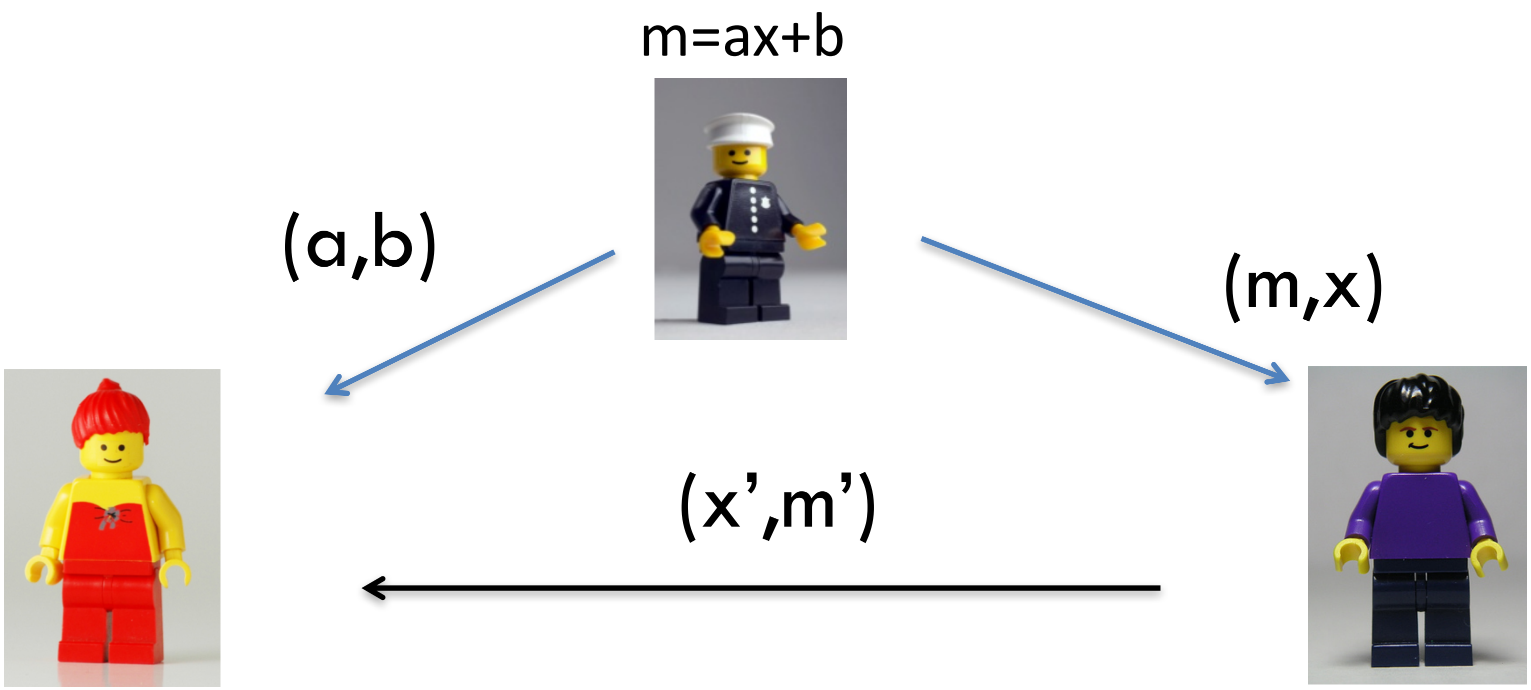
$T2[u,v] + e2$ ←

Is this cheating?

- $v = y + s + e1 = (y+e1) + s = y' + s$
  - Input substitution, not cheating according to the definition!
- $M2[u,v] + e2$
  - Changes output to $z' = f(x,y) + e2$
  - Example: $f(x,y)=1$ iff $x=y$ *(e.g. pwd check)*
  - $e2=1$ the output is 1 whp *(login without pwd!)*
    - *Clearly breach of security!*

Force Bob to send the right value

- **Problem:** Bob can send the wrong shares
- **Solution:** use MACs
  - e.g. m=*ax+b with (a,b)* $\leftarrow$ *F*



Abort if m’≠ax’+b

OTTT+MAC

T1, r A B

T2, s M

u = x + r

v = y + s

T2[u,v], M[u,v]

If (M[u,v]=A[u,v]*T2[u,v]+B[u,v])

output f(x,y) = T1[u,v] + T2[u,v]

else

abort

Statistical security vs. malicious Bob w.p. 1-1/|F|

“The simplest 2PC protocol ever” OTTT

- **Optimal communication complexity ☺**
- **Storage exponential in input size ☹**

➔Represent function using circuit instead of truth table!

Part 1: Secure Computation with a Trusted Dealer

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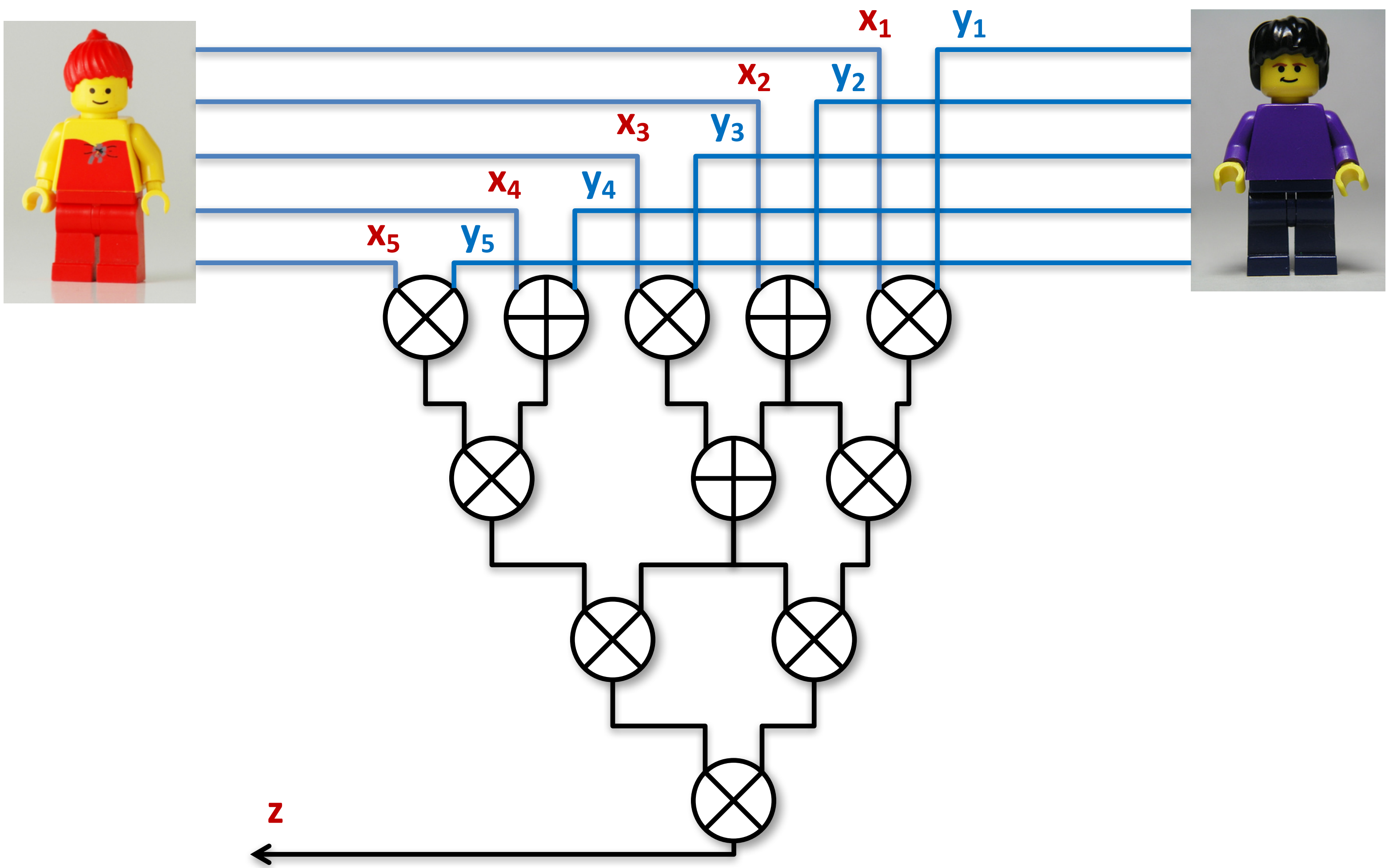
Circuit based computation



Invariant

- For each ***wire x*** in the circuit we have
  - $[x] := (x_A, x_B)$ *// read "x in a box"*
  - Where Alice holds x_A
  - Bob holds x_B
  - Such that $x_A+x_B=x$

- Notation overload:
  - x is both the r-value and the l-value of x
  - use n(x) for name of x and v(x) for value of x when in doubt.
  - Then $[n(x)] = (x_A,x_B)$ such that $x_A+x_B=v(x)$

Circuit Evaluation (Online phase)

1) [x] ← Input(A,x) :

- chooses random x_B and send it to Bob
- set x_A=x+x_B // symmetric for Bob

Alice only sends a random bit! “Clearly” secure

2) z ← Open(A,[z]): // z← Open([z]) if both get output

- Bob sends z_B
- Alice outputs z=z_A+z_B // symmetric for Bob

Alice should learn z anyway! “Clearly” secure

Circuit Evaluation (Online phase)

2) [z]← Add([x],[y]) // at the end z=x+y

- Alice computes $z_A = x_A + y_A$
- Bob computes $z_B = x_B + y_B$

- We write [z] = [x] + [y]

No interaction! “Clearly” secure

“for free” : only a local addition!

Circuit Evaluation (Online phase)

2a) [z]← Mul(a,[x]) // at the end z=a*x

- Alice computes $z_A = a * x_A$
- Bob computes $z_B = a * x_B$

2c) [z]← Add(a,[x]) // at the end z=a+x

- Alice computes $z_A = a + x_A$
- Bob computes $z_B = x_B$

Circuit Evaluation (Online phase)

3) Multiplication?

How to compute [z]=[xy] ?

How do we compute this?

Alice, Bob should compute

$$z_A + z_B = (x_A+x_B)(y_A+y_B)$$
$$= x_A y_A + x_B y_A + x_A y_B + x_B y_B$$

Alice can compute this

Bob can compute this

Circuit Evaluation (Online phase)

3) [z]←Mul([x],[y]):

1. Get *[a],[b],[c]* with *c=ab* from trusted dealer
2. *e=Open([a]+[x])*
3. *d=Open([b]+[y])*

Is this secure?
e,d are “one-time-pad” encryptions of x and y using a and b

4. Compute [z] = [c] + e[y] + d[x] - ed

$$ab + (ay+xy) + (bx+xy) - (ab+ay+bx+xy)$$

Part 1: Secure Computation with a Trusted Dealer

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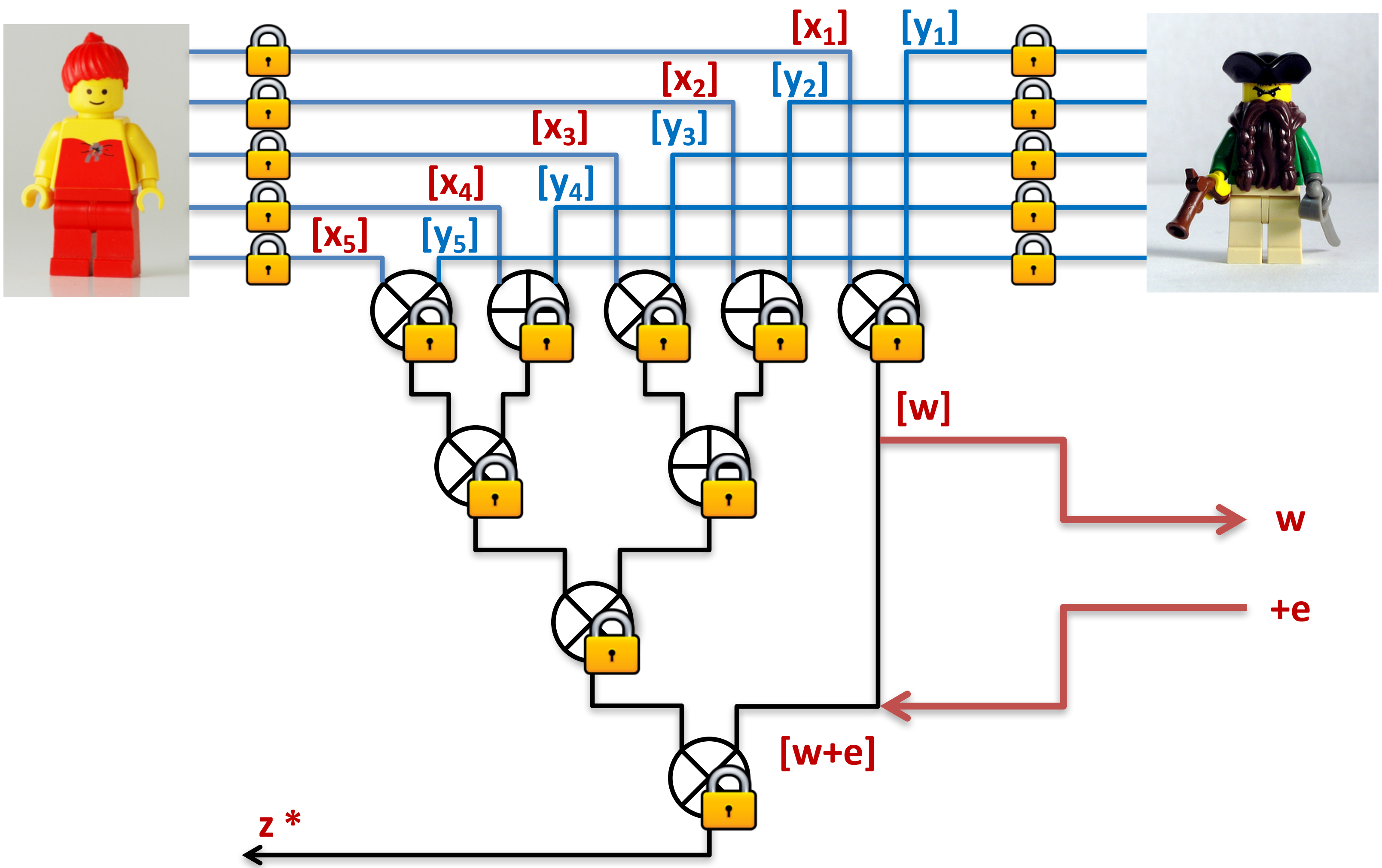
Secure Computation



Active Security?

- **“Privacy?”**
  – even a malicious Bob does not learn anything ☺

- **“Correctness?”**
  – a corrupted Bob can change his share during any “Open” (both final result or during multiplication) leading the final output to be incorrect ☹

Problem

2) z ← Open(A,[z]):

- Bob sends $z_B + e$
- Alice outputs $z = z_A + z_B + e$ // error change output distribution in way that cannot be simulated by input substition

Solution: add MACs

2) $z \leftarrow$ Open(A,[z]):

- Bob sends z_{B}, m_B
- Alice outputs
  - $z = z_A + z_B$ if $m_B = z_B \Delta_A + k_A$
  - "abort" otherwise

- **Solution:** Enhance representation [x]
  - $[x] = ((x_A, k_A, m_A), (x_B, k_B, m_B))$ s.t.
  - $m_B = x_B \Delta_A + k_A$ (symmetric for m_A)
  - Δ_A, Δ_B is the same for all wires.

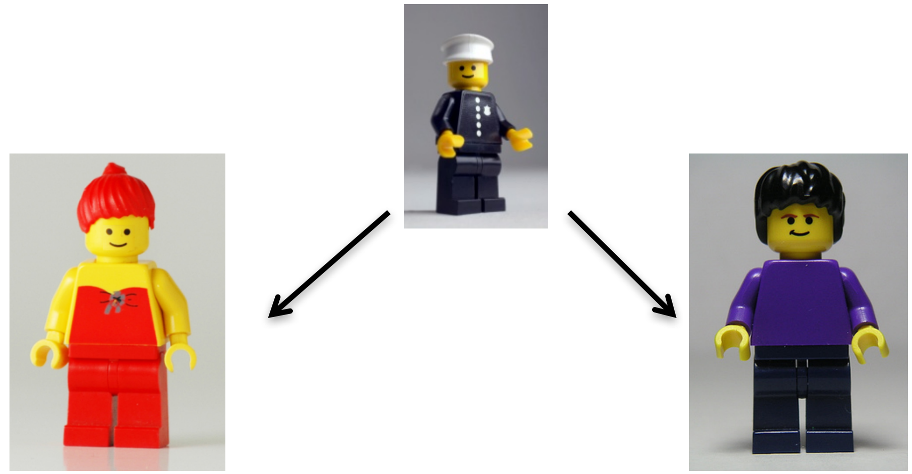
Linear representation

- Given
  - $[x] = ((x_A, k_{Ax}, m_{Ax}) , (y_B, k_{Bx}, m_{Bx}))$
  - $[y] = ((y_A, k_{Ay}, m_{Ay}) , (y_B, k_{By}, m_{By}))$
  - Compute $[z] = ($
    $(z_A = x_A + y_A \quad , \qquad k_{Az} = k_{Ax} + k_{Ay} \quad , \quad m_{Az} = m_{Ax} + m_{Ay}) ,$
    $(z_B = x_B + y_B \quad , \qquad k_{Bz} = k_{Bx} + k_{By} \quad , \quad m_{Bz} = m_{Bx} + m_{By}) ,)$

- And [z] is in the right format since…

$$m_{Bz} = (m_{Bz} + m_{By}) = (k_{Ax} + x_B \Delta_A) + (k_{Ay} + y_B \Delta_A)$$
$$= (k_{Ax} + k_{Ay}) + (x_B + y_B)\Delta_A = k_{Az} + z_B \Delta_A$$

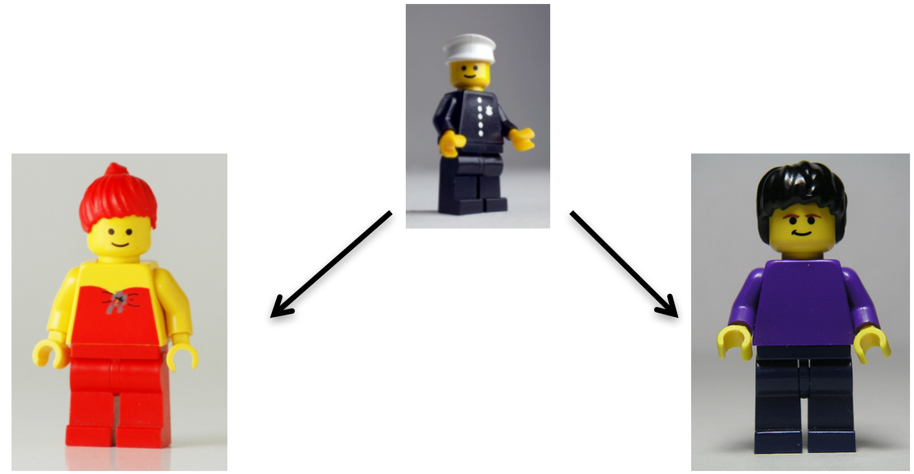
Recap



1. **Output Gates:**
   - Exchange shares and MACs
   - Abort if MAC does not verify
2. **Input Gates**:
   - Get a random [r] from ***trusted dealer***
   - r ← Open(A,[r])
   - Alice sends Bob *d=x-r*,
   - Compute [x]=[r]+d

Allows simulator to extract x* = r+d*

Recap



1. **Addition Gates:**
   – Use linearity of representation to compute

$$[z]=[x]+[y]$$

2. **Multiplication gates:**
   – Get a random triple [a][b][c] with c=ab from 
   – e ← Open([a]+[x]), d ← Open([b]+[y])
   – Compute [z] = [c] + a[y] + b[x] - ed

Final remarks

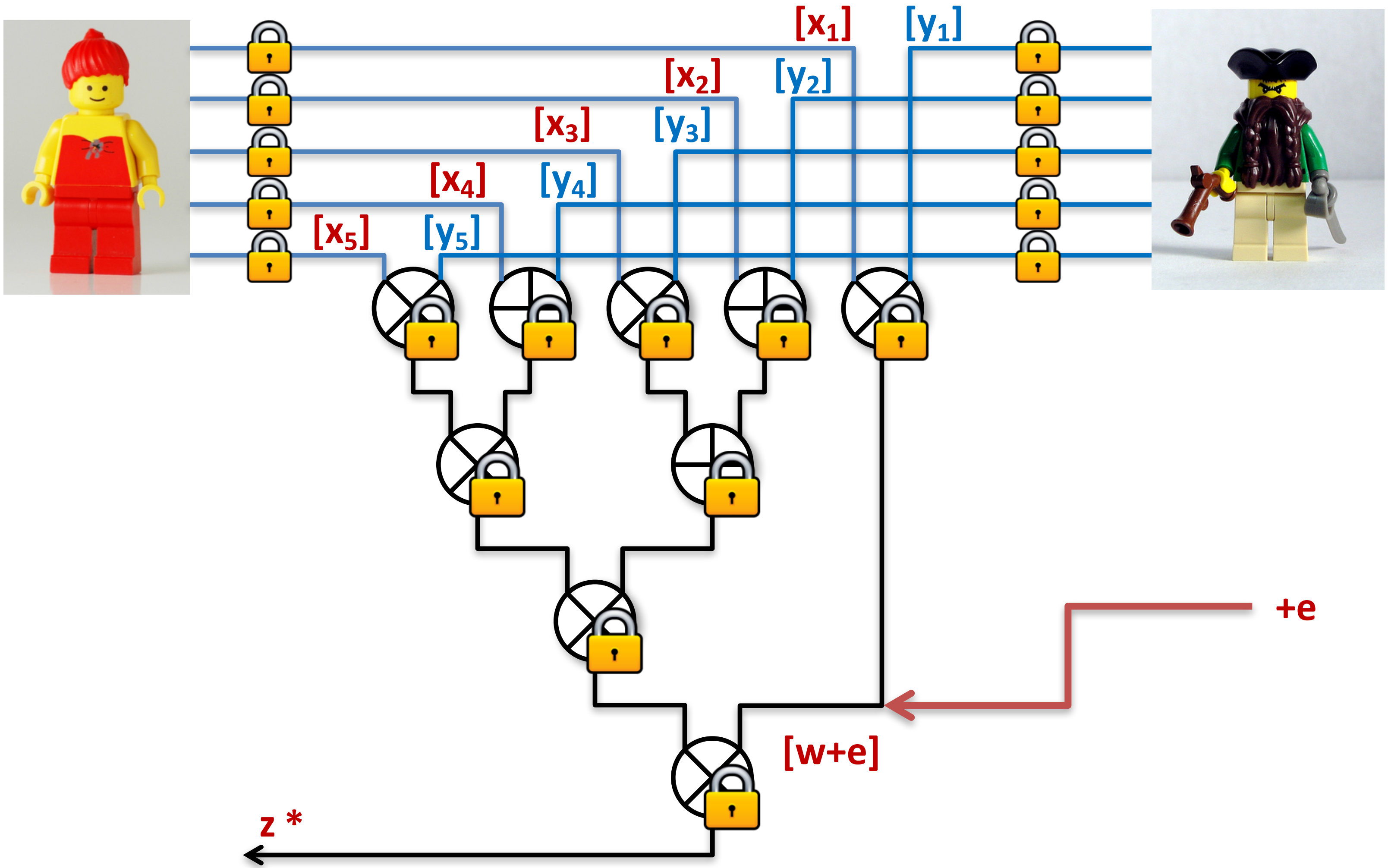
- Size of MACs
- Lazy MAC checks

Size of MACs

1. Each party must store a mac/key pair ***for each other party***
   - quadratic complexity! ☹
   - SPDZ for linear complexity.

2. MAC is only as hard as guessing key!
   k MACs in parallel give security $\boldsymbol{1/|F|^k}$
   - In *TinyOT* $\mathbf{F=Z_2}$, then MACs/Keys are *k-bit strings*
   - *MiniMACs* for constant overhead

Lazy MAC Check



Lazy MAC Check

1) $z \leftarrow$ PartialOpen(A,[z]):

1. Bob sends z_B
2. Bob runs OutMAC.append(m_B)
3. Alice runs InMAC.append($k_A + z_B \Delta_A$)
4. Alice outputs $z=z_A+z_B$

2) $z \leftarrow$ FinalOpen(A,[z]):

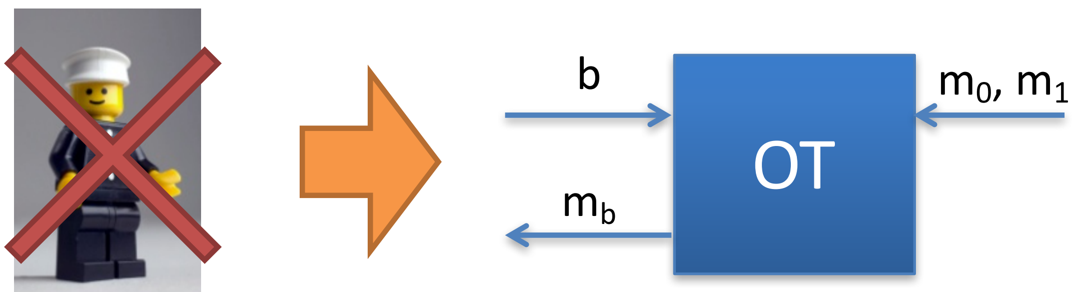
1. Steps 1-3 as before
2. Bob sends u=H(OutMAC) to Alice
3. Alice outputs $z=z_A+z_B$ if u=H(InMAC)
4. "abort" otherwise

Recap of Part 1

- Two protocols *“in the trusted dealer model”*
  - **One Time-Truth Table**
    - **Storage** exp(input size) ☹
    - **Communication** O(input size) ☺
    - 1 **round** ☺
  - **(SPDZ)/BeDOZa/TinyOT online phase**
    - **Storage** linear #number of AND gates
    - **Communication** linear #number of AND gates
    - **#rounds** = depth of the circuit
  - …and add enough **MACs** to get **active security**

Recap of Part 1

- To do secure computation is enough to precompute enough **random multiplications**!



- If no ***semi-trusted party is available***, we can use **cryptographic assumption** (next)